The evolution equation for the OTD modes which I want to solve is

$$\frac{\partial \mathbf{u}}{\partial t} = L_{\text{NS}}(\mathbf{u}) - \langle L_{\text{NS}}(\mathbf{u}), \mathbf{u} \rangle \mathbf{u}$$

Where L_{NS} is the linearized Navier-Stokes operator, given by

$$L_{\text{NS}}(\mathbf{u}) = -(\mathbf{U}_b.\nabla)\mathbf{u} - (\mathbf{u}.\nabla)\mathbf{U}_{\text{b}} + \frac{1}{\text{Re}}\nabla^2\mathbf{u} - \nabla p$$

and the inner product $\langle .,. \rangle$ is the standard energy norm over the computational domain.

I know from the User Guide that Nektar++ solves the linearized NS equation:

$$\frac{\partial \mathbf{u}}{\partial t} = L_{\text{NS}}(\mathbf{u})$$

I think the additional inner product term $\langle L_{\text{NS}}(\mathbf{u}), \mathbf{u} \rangle \mathbf{u}$ can be introduced directly via the external forcing term.

$$\frac{\partial \mathbf{u}}{\partial t} = L_{\text{NS}}(\mathbf{u}) + \mathbf{f}$$

where $\mathbf{f} = -\langle L_{\text{NS}}(\mathbf{u}), \mathbf{u} \rangle \mathbf{u}$

Does Nektar++ accept this forcing term for the linearized NS? I know Nek5000 can do it.

The next issue is how to calculate this forcing term in each time step from the flow values in the previous time step.

Does Nektar++ have a tool to calculate this inner product term $\langle L_{\text{NS}}(\mathbf{u}), \mathbf{u} \rangle \mathbf{u}$? How can I compute the action of the linearized NS operator on the perturbation field?