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Aim: solve the dimensionless incompressible Navier-Stokes

$$\frac{\partial \boldsymbol{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \boldsymbol{v} = -\nabla p + \frac{1}{Re} \nabla \cdot \left(2\boldsymbol{D} - \frac{2}{3} (\nabla \cdot \boldsymbol{v}) \boldsymbol{I} \right) \tag{1a}$$

$$\nabla \cdot \boldsymbol{v} = 0 \tag{1b}$$

$$\boldsymbol{D} = \frac{1}{2} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \tag{2}$$

REMARK: I know that due to incompressibility the viscous stress term on the right hand side reduces to $\frac{1}{Re} \nabla^2 \boldsymbol{v}$ but I want to keep it as it is right know, as I will further build on that system

1.1 Weak formulation

$$\int_\Omega \left(\boldsymbol{\chi} \cdot \left(\frac{\partial \boldsymbol{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \boldsymbol{v} \right) - p(\nabla \cdot \boldsymbol{\chi}) + \frac{2}{Re} \boldsymbol{D} : \nabla \boldsymbol{\chi} - \frac{2}{3Re} (\nabla \cdot \boldsymbol{v})(\nabla \cdot \boldsymbol{\chi}) \right) \mathrm{d}\Omega = 0 \tag{3a}$$

$$\int_\Omega (\nabla \cdot \boldsymbol{v}) \, \theta \, \mathrm{d}\Omega = 0 \tag{3b}$$

I think the problem lies in the way I implement $\boldsymbol{D}$ in Firedrake, which is:

$2\boldsymbol{D} : \nabla \boldsymbol{\chi} = \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \frac{\partial \chi_i}{\partial x_j} =$ `inner((nabla_grad(v)[i,j]+nabla_grad(v)[j,i]),nabla_grad(chi)[i,j])`

this gives me L_2 norm of 0.00677501099488 and verical flow near outlet...

If I manually delete $\frac{\partial v_j}{\partial x_i}$ term so that I'm left with:

$2\boldsymbol{D} : \nabla \boldsymbol{\chi} \rightarrow$ `inner(nabla_grad(v)[i,j],nabla_grad(chi)[i,j])`

I get L_2-norm of $5.86838100481 \times 10^{-14}$