The following equation needs to be solved for $\mu_h^{n+1/2}$; $\delta\mu_h$ acts as the test function and everything on the RHS is known. Function $\Theta(x-L_p)$ is a Heaviside function and is zero for $x < L_p$ and one for $x \geq L_p$.

$$\begin{aligned}
&\int_0^L \Theta(x-L_p)\,\partial_x\delta\mu_h\partial_x\mu_h^{n+1/2}\,\mathrm{d}x + \frac{\rho}{M}\int_0^L \Theta(x-L_p)\,\delta\mu_h\,\mathrm{d}x\int_0^L \Theta(x-L_p)\,\mu_h^{n+1/2}\,\mathrm{d}x \\
&\quad = \frac{1}{\Delta t}\left(\tilde{Z}^n\int_0^L \Theta(x-L_p)\delta\mu_h\,\mathrm{d}x - \int_0^L \Theta(x-L_p)\,\delta\mu_h\eta_h^n\,\mathrm{d}x\right) + W^n\int_0^L \Theta(x-L_p)\delta\mu_h\,\mathrm{d}x \\
&\qquad + \int_0^L \left(H_0\Theta(L_p-x) + H_b(x,\bar{Z})\Theta(x-L_p)\right)\partial_x\delta\mu_h\big(\frac{g\Delta t}{2}\partial_x\eta_h^n - \partial_x\phi_h^n\big)\,\mathrm{d}x
\end{aligned}$$