Firedrake: extruded meshes and outer-product elements

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At the time of writing, FEniCS only supports fully unstructured meshes with simplicial elements – intervals, triangles and tetrahedra. There are many examples where a partially-structured mesh, with non-simplicial cells, is more appropriate. This is especially true if the domain (or the PDE) is far from isotropic. A classic example is in geophysical simulations, where the vertical thickness of the atmosphere or ocean is several orders of magnitude smaller than the horizontal scales.

In Firedrake, we label these meshes *extruded*: a 'base' mesh is extruded upwards or outwards to form a new mesh with a regular layered structure. This partial structure also helps to offset the efficiency losses that arise from using an unstructured base mesh. The geometric cells become the *product* of the 'base' triangular or interval cell with an interval: a triangular prism or quadrilateral, respectively.

We also require finite element spaces on such cells. At a local level, these can often be expressed in terms of *products* of existing elements. The simplest example is the CG element on a prism. This is just the *outer product* of the CG elements on a triangle and on an interval. However, far more unusual finite element spaces can also be constructed in the same way, including those compatible with the principles of FEEC[1].

In this talk, we will give some details of how extruded meshes are implemented within Firedrake, and some of the modifications we made to FIAT and FFC in order to automatically generate these new types of elements.

References

[1] Douglas N. Arnold and Richard S. Falk and Ragnar Winther. Finite element exterior calculus, homological techniques, and applications. *Acta Numerica* , Vol. **15**, 1–155, 2006.