Initial-Boundary-Value problem

p_w-S_n-formulation

$$\frac{\partial(\phi\rho_w(1-S_n))}{\partial t} + \operatorname{div}\left(\rho_w \frac{k_{rw}}{\mu_w}\mathbb{K}(\nabla p_w - \rho_w \mathbf{g})\right) = \rho_w q_w$$

$$\frac{\partial(\phi\rho_n S_n)}{\partial t} + \operatorname{div}\left(\rho_n \frac{k_{rn}}{\mu_n}\mathbb{K}(\nabla p_w + \nabla p_c - \rho_n \mathbf{g})\right) = \rho_n q_n$$

Initial conditions

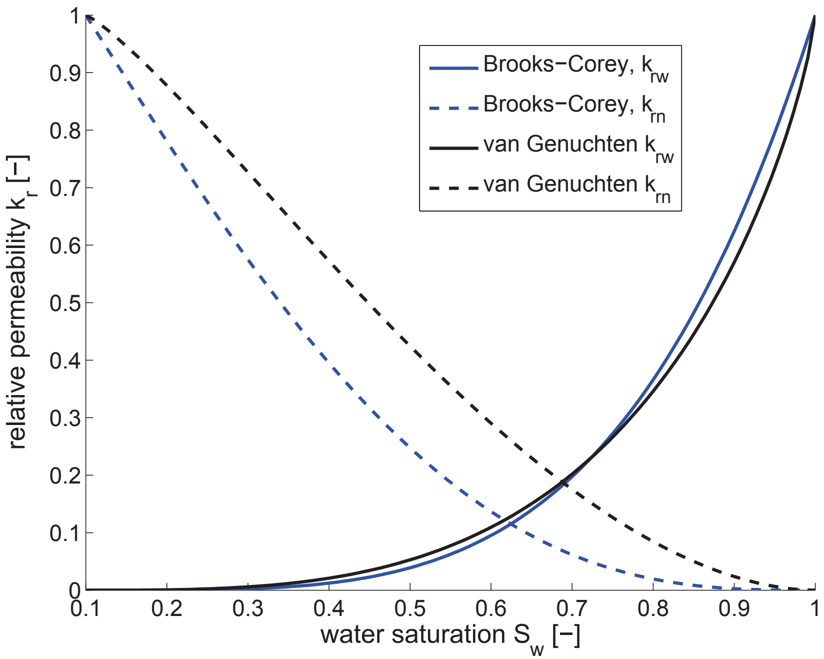
$$S_n(\mathbf{x},0) = S_{n0}(\mathbf{x}), \qquad p_w(\mathbf{x},0) = p_{w0}(\mathbf{x}) \quad \mathbf{x} \in \Omega$$

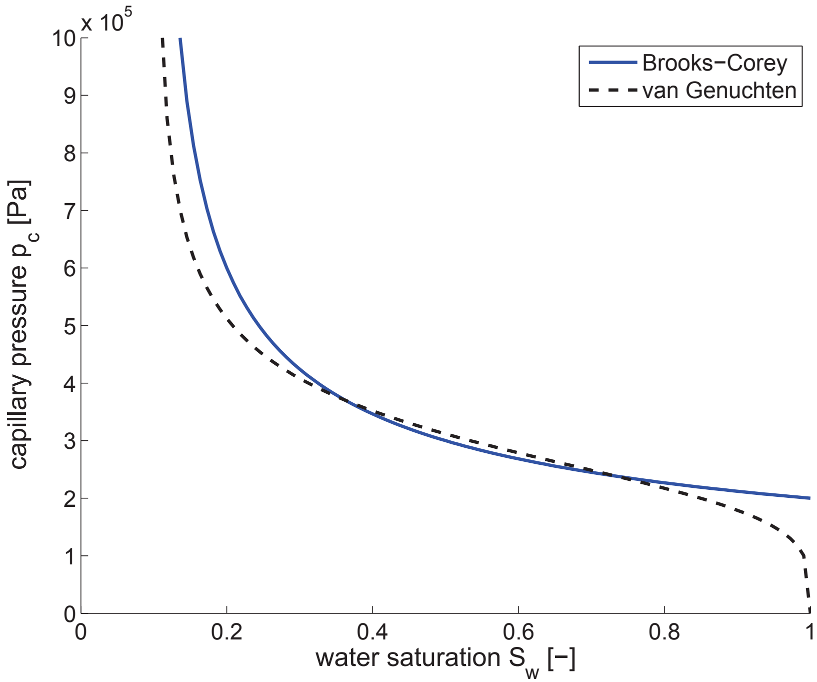
Boundary conditions

$$p_w(\mathbf{x},t) = g_{Dw}(\mathbf{x},t) \text{ on } \Gamma_{Dw} \qquad \rho_w \mathbf{v}_w \cdot \mathbf{n} = g_{Nw}(\mathbf{x},t) \text{ on } \Gamma_{Nw}$$

$$S_n(\mathbf{x},t) = g_{Dn}(\mathbf{x},t) \text{ on } \Gamma_{Dn} \qquad \rho_n \mathbf{v}_n \cdot \mathbf{n} = g_{Nn}(\mathbf{x},t) \text{ on } \Gamma_{Nn}$$

Nonlinearities





Brooks-Corey

$$k_{rw} = S_e^{\frac{2+3\lambda}{\lambda}}$$

$$k_{rn} = (1 - S_e)^2 \left(1 - S_e^{\frac{2+\lambda}{\lambda}}\right)$$

van Genuchten

$$k_{rw} = \sqrt{S_e}\left(1 - (1 - S_e^{1/m})^m\right)^2$$

$$k_{rn} = (1 - S_e)^{\frac{1}{3}} \left(1 - S_e^{\frac{1}{m}}\right)^{2m}$$

Brooks-Corey

$$p_c = p_d S_e^{-1/\lambda}$$

van Genuchten

$$p_c = \frac{1}{\alpha}(S_e^{-1/m} - 1)^{1/n}$$

Effective saturation

$$S_e = \frac{S_w - S_{wr}}{1 - S_{wr} - S_{nr}}, \quad 0 \leq S_e \leq 1$$