

The following three equations need to be solved simultaneously for $\phi^{n+1/2}$, $\eta^{n+1/2}$ and $\lambda^{n+1/2}$. The weak formulation for Firedrake has the following form and includes the use of a Heaviside step function $\Theta(x - L_p)$ and three test functions $v_{1,2,3}$,

$$\int_0^L v_1 \left(\phi^{n+1/2} - \phi^n + \frac{\Delta t}{2} g \eta^n - \Theta(x - L_p) \lambda^{n+1/2} \right) dx = 0 \quad (1a)$$

$$\int_0^L \left(v_2 (\eta^{n+1} - \eta^n) - \Delta t H(x) \partial_x v_2 \partial_x \phi^{n+1/2} \right) dx = 0 \quad (1b)$$

$$\int_0^L v_3 \Theta(x - L_p) \left(\frac{1}{\Delta t} (\eta^{n+1} - Z^n) - W^n + \frac{\rho}{M} \int_0^L \Theta(x - L_p) \lambda^{n+1/2} dx \right) dx = 0. \quad (1c)$$

Question: On attempting to solve to problem as a mixed system using Schur complements, how to treat the last term?