surface_ mesh = RectangleMesh(Nx,Ny,Lx,Ly)
mesh = ExtrudedMesh(surface_ mesh, layers=Nz, layer_ height=d_ x, extrusion_ type='uniform')
V=FunctionSpace(mesh,"CG",1)
W=V*V
q,v=Testfunction(W)
w=Function(W)

Then we define ϕ_s, which is the value of ϕ at the surface, and φ which is the difference between ϕ^*, the solution for ϕ in the 3D domain, and ϕ_s : $\varphi = \phi^* - \phi_s$.

ϕ_s, φ = split(w)
$\phi^* = \phi_s + \varphi$

Both ϕ_s and ϕ^* are equal to 0.0 initially.

The objective is to solve $R\_phi\_s + R\_phi\_star = 0$, with boudary condition $\varphi = \phi^* - \phi_s = 0$ at $\sigma = H_0$.
R_phi_s and R_phi_star are defined below:

$$R\_phi\_s = q * \left(\frac{\phi_s^{n+1/2} - \phi_s^n}{dt/2} + g(h^n - H_0) + \frac{1}{2}(\nabla\phi_s^{n+1/2})^2 - \frac{1}{2}\left(\frac{H_0}{h^n}\right)^2 (\partial_\sigma\phi^*)^2 \left(1 + (\nabla h^n)^2\right)\right) * ds\_t \quad (1)$$

$$\begin{aligned} R\_phi\_star = &\Bigg\{ -\nabla\phi^*\nabla v + \frac{\sigma}{h^n}\nabla h^n \nabla v \partial_\sigma \phi^* + \frac{1}{h^n} v \nabla h^n \nabla\phi^* + \frac{\sigma}{h^n}\nabla\phi^*\nabla h^n \partial_\sigma v - v\frac{\sigma}{h^{n2}}(\nabla h^n)^2\partial_\sigma\phi^* \\ &- \left[\left(\frac{\sigma}{h^n}\nabla h^n\right)^2 + \left(\frac{H_0}{h^n}\right)^2\right]\partial_\sigma\phi^*\partial_\sigma v \Bigg\} * dx \\ &+ v\left[\left(\frac{H_0}{h^n}\right)^2\left[1 + (\nabla h^n)^2\right]\partial_\sigma\phi^* - \frac{H_0}{h^n}\nabla h^n \nabla\phi^*\right]_{\sigma = H_0} * ds\_t \end{aligned} \quad (2)$$

As a reminder, ∇ is only the horizontal gradient $(\partial_x, \partial_y)^T$, and ∂_σ is the vertical derivative ($dx(2)$).
Question: ϕ_s is updated only at the surface $z = H_0$, but is defined in the 3D domain. Therefore, the interior nodes of ϕ_s are always equal to the initial value, that is 0.0. This means that $\varphi = \phi^* - \phi_s$ gets the wrong value in the interior. How can I copy the surface value of ϕ_s in its interior nodes? The issue is that it needs to be done while solving the equations. A solution could be to add a term in the residual, forcing $\phi_s(x, y, z)$ to be equal to $\phi_s(x, y, H_0)$, e.g.:

$$q * phi_s^{n+1/2} * dx - q * phi_s^{n+1/2} * ds\_t \quad (3)$$

, but in that case, which test function should I use for this part of the residual? Is it correct to use q ? Is there another/a better way to update the interior of ϕ_s while solving the equation?