The fluid domain is $\Omega = [0, L]^2 \setminus \Omega_{\text{beam}}$ (basically a 2D region with a hole representing the beam). Let us define the velocity field

$$\mathbf{u} = \mathbf{u}_0 + \hat{\mathbf{u}},$$

$\mathbf{u}_0 = 0$ in $\partial\Omega_{\text{beam}}$ and $\hat{\mathbf{u}}$ is introduced to account for the coupling between the fluid and the 1D beam which is discretized with Hermite elements. Let us decompose

$$\hat{\mathbf{u}} = \sum_{i=1}^{n} s_i \, \boldsymbol{\phi}_i$$

in which $\boldsymbol{\phi}_i$ is zero everywhere except in $\Gamma_i \subset \partial\Omega_{\text{beam}}$ and s_i are the velocities of the n beam degrees of freedom.

The problem reads: Find $\mathbf{u_0} \in V_0$, $p \in Q$, $\ell \in \mathbb{R}$, $s_i \in \mathbb{R}$, $i = 1, \ldots, n$, such that

$$\begin{aligned}
\int_\Omega 2\mu(\mathbf{u})\nabla^S\mathbf{u} : \nabla\mathbf{v} - \int_\Omega p\nabla\cdot\mathbf{v} - \int_\Omega \mathbf{f}\cdot\mathbf{v} &= 0, \quad \forall \mathbf{v} \in V_{00} && (1)\\
\int_\Omega q\nabla\cdot\mathbf{u} + \ell\int_\Omega q \ + \ \text{stabilization} &= 0, \quad \forall q \in Q && (2)\\
r\int_\Omega p &= 0, \quad r \in \mathbb{R} && (3)\\
\underbrace{\left[\int_\Omega 2\mu(\mathbf{u})\nabla^S\mathbf{u} : \nabla\boldsymbol{\phi}_i - \int_\Omega p\nabla\cdot\boldsymbol{\phi}_i - f^i_{\text{beam}}\right]}_{\delta\mathcal{W}_{\text{viscous}} - \delta\mathcal{U}_{\text{elastic}}} &= 0, \quad i = 1, \ldots, n && (4)
\end{aligned}$$

where f^i_{beam}'s are given data. They represent the elastic forces in the beam which at this point are considered frozen at the previous iteration.