Firedrake: a multilevel domain specific language approach to unstructured mesh stencil computations

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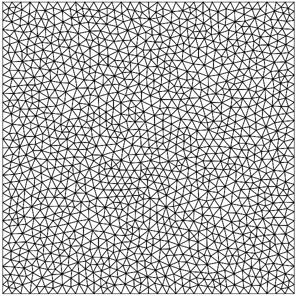
What are we interested in?

- (Predominantly) finite element simulations
  - primary application areas in geophysical fluids (ocean and atmosphere)
  - simulations on unstructured and semi-structured meshes
- Providing high-level interfaces for users, with performance

  - the moon, on a stick

How does FE fit a stencils session?

- Numerics tells us the elementary operation we apply everywhere in the mesh (a "kernel")
- Mesh topology gives us the "stencil" pattern
- Our job: efficiently apply the kernel over the whole mesh



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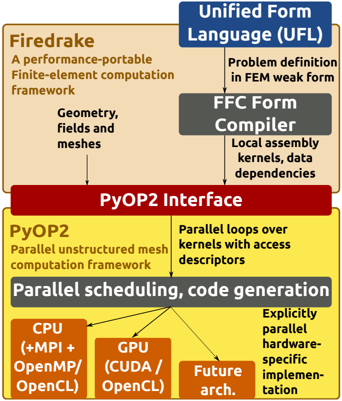
Express what, not how

- User code should make as few decisions about implementation as possible
- FE discretisations expressed symbolically using the Unified Form Language
    - developed in the FEniCS project (`http://www.fenicsproject.org`)
    - symbolic representation compiled to a C kernel
- Data to feed to kernel (and interface to solvers) provided by Firedrake (`http://www.firedrakeproject.org`)
- Execution of kernel over entire domain expressed as parallel loop with access descriptors
    - uses PyOP2 unstructured mesh library (`http://github.com/OP2/PyOP2`)
    - implementation of loop taken out of user hands

Firedrake

- High level finite element computations
  - `http://www.firedrakeproject.org`

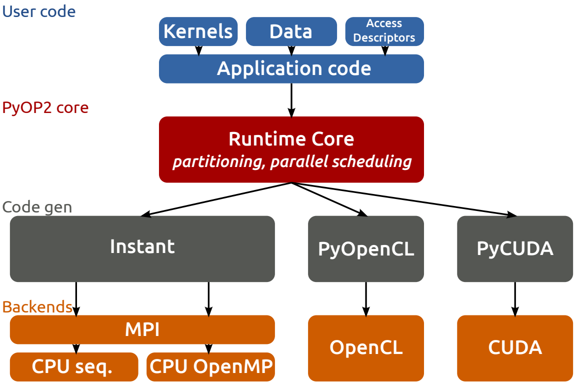


An example

```
from firedrake import *
mesh = UnitSquareMesh(32, 32)
BDM = FunctionSpace(mesh, "BDM", 3)
DG = FunctionSpace(mesh, "DG", 2)
W = BDM * DG
sigma, u = TrialFunctions(W)
tau, v = TestFunctions(W)
f = Function(DG).interpolate(Expression(
    "10*exp(-(pow(x[0] - 0.5, 2) + pow(x[1] - 0.5, 2)) / 0.02)"))
a = (dot(sigma, tau) + div(tau)*u + div(sigma)*v)*dx
L = - f*v*dx
bc0 = DirichletBC(W.sub(0), Expression(("0.0", "-sin(5*x[0])")), 1)
bc1 = DirichletBC(W.sub(0), Expression(("0.0", "sin(5*x[0])")), 2)
w = Function(W)
solve(a == L, w, bcs=[bc0, bc1])
sigma, u = w.split()
```

PyOP2

- A python library for unstructured mesh computations
  - `http://github.com/OP2/PyOP2`



PyOP2 data model

- Data types

Set	e.g. cells, degrees of freedom (dofs)
Dat	data defined on a `Set` (one entry per set element)
Map	a mapping between two sets (e.g. cells to dofs)
Global	global data (one entry)
Kernel	a piece of code to execute over the mesh

- access descriptors
  - READ, RW, WRITE, INC
- iteration construct

`par_loop`	execute a Kernel over every element in a `Set`

Example

```
par_loop(kernel, elements,
         element_data(READ),  # direct access
         node_data(WRITE, elem_node), # indirect
         count(INC))
```

- executes kernel for each ele in elements

  first argument data corresponding to ele (read-only)

  second argument the nodes of this ele (written)

  third argument a global counter (incremented)
- runtime knows it has to care about data dependencies for
  - write to node_data
  - increment into elem_count

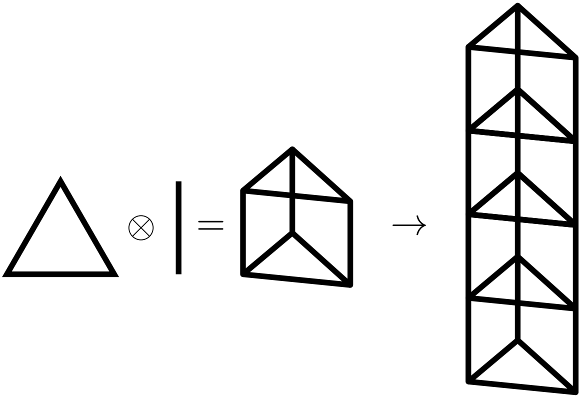
Synthesis, not analysis

- Access descriptors on parallel loops mean:
  - code generation requires synthesis, not analysis
  - determination of when halo exchanges need to occur is automatic
  - colouring for shared memory parallelisation can be computed automatically

Semi-structured meshes

- Many application areas have a "short" direction
  - ocean and atmosphere
  - thin shells
- Numerics dictate we should do something different in short direction
- Use semi-structured meshes
  - unstructured in "long" directions, structured in short
  - can we exploit this structure?

A picture of triangles



Admits a fast implementation

- Exploit structure in mesh to amortize indirect lookups
  - arrange for iteration over short direction to be innermost loop
  - pay one indirect lookup per mesh column
  - walk up column directly

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A bandwidth bound test case

- Walk over mesh, read from vertices and cells, sum into global

```c
void kernel(double *a, double *x[], double *y[]) {
    const double area = fabs(x[0][0]*(x[2][1]-x[4][1])
                             + x[2][0]*(x[4][1]-x[0][1])
                             + x[4][0]*(x[0][1]-x[2][1]));
    *a += area * 0.5 * y[0][0];
}
```

- Can we sustain an appreciable fraction of memory bandwidth?

Measuring throughput

- "Effective" data volume
  - assume every piece of data is touched exactly once (in perfect order)
  - don't count data movement for indirection maps
  - effectively, just count the volume of degrees of freedom touched
- "Valuable" bandwidth
  - effective data volume per second
- Actual memory bandwidth will be higher (reading indirection maps)
  - but this is not "useful"

Benchmark setup

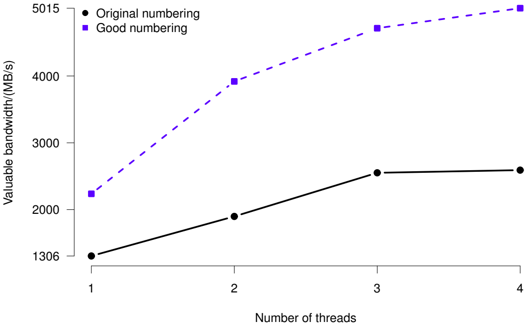
- 2D unstructured mesh: 806110 cells, 403811 vertices.
  - 2D coordinate field located at vertices (implicit 3rd coordinate)
  - scalar field stored at cell centres
- Run with increasing number of extruded cell layers (n_{layer})
  - data volume ($806110 * n_{layer}$) + $403811 * 2$ ($n_{layer} + 1$) doubles
  - 1 layer: 18.4MB
  - 200 layers: 2468MB
- Execute kernel over mesh 100 times

Single node

- Intel Sandybridge 4 cores (2 way hyperthreading)
  - 32kB L1 cache (per core)
  - 256 kB L2 cache (per core)
  - 8 MB L3 cache (shared)
- Measured stream bandwidth (8 threads)
  - 11341 MB/s

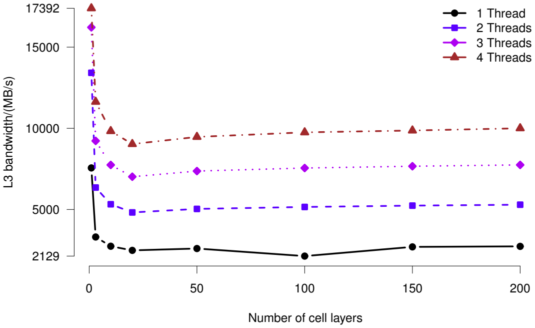
Effect of good base numbering

- Being completely unstructured hurts a lot
- Compare default (mesh generator) numbering with renumbered mesh using 2D space filling curve



Adding layers amortizes indirection cost

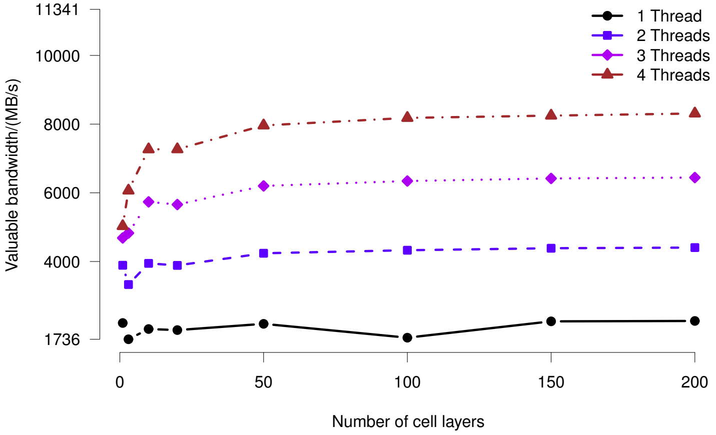
- L3 cache bandwidth
  - low layer numbers hit the L3 more often (indirection lookups)



- What about actual throughput though?

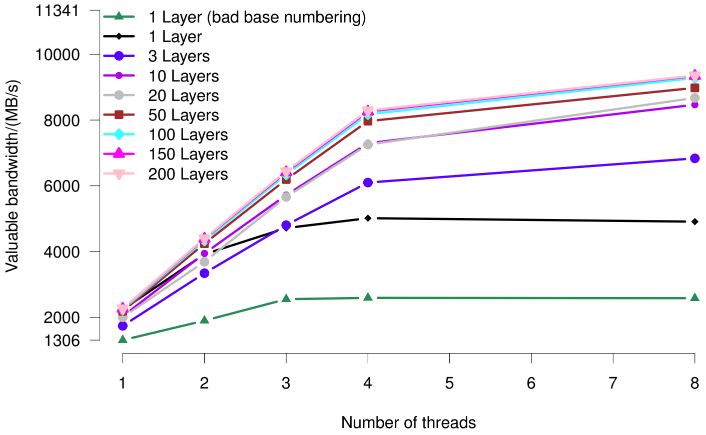
Valuable bandwidth

- Above ~20 layers, indirection cost "hidden"



More threads

- Hyperthreading gives some further gains (82% stream bandwidth)



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Possible to be unstructured and fast

- A good numbering gets you a reasonable way there
- If there is structure in your problem, use it!
- High level abstractions need not kill performance

Thanks

- Institutions
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